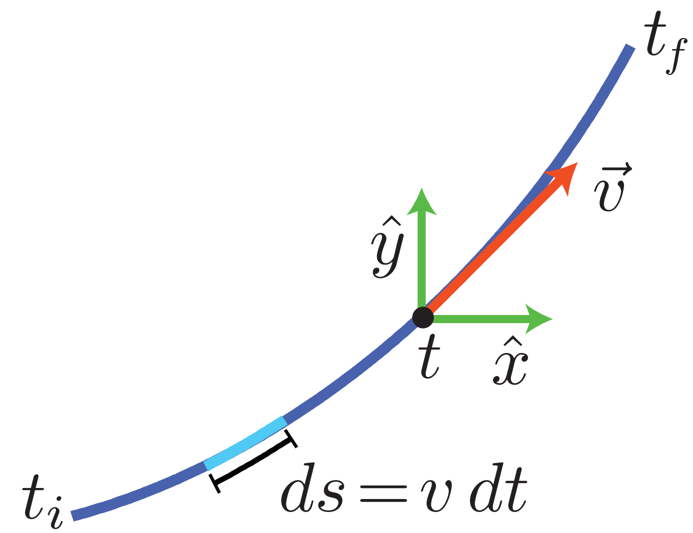
What’s special about the coins?

Tea talk
Huh?

curve representation

$$\Gamma : \mathbb{I} \subset \mathbb{R} \to \mathbb{R}^2$$

$$\Gamma(t) = (x(t), y(t))$$



- Velocity

$$\vec{v} = \vec{\Gamma}'(t)$$

- Curve reconstruction

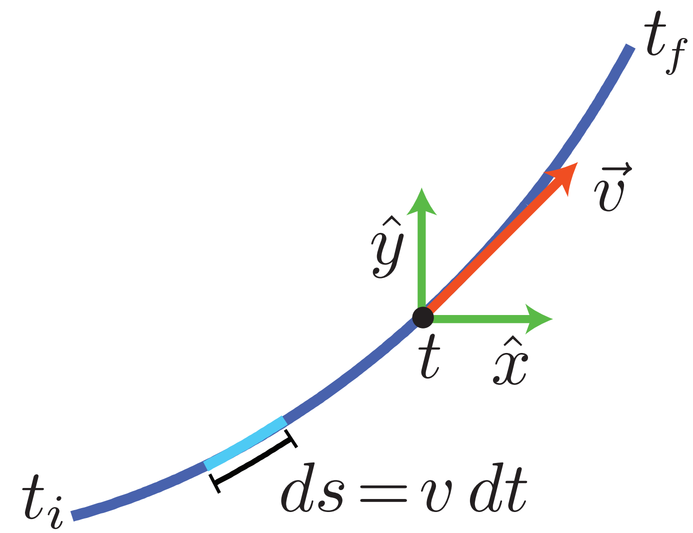
$$\Gamma(t) = \Gamma(t_o) + \int_{t_o}^{t} \vec{v}(\tilde{t})\, d\tilde{t}$$

Natural parameterization

- Arc-length coordinate

$$\Gamma(s) = (x(s), y(s))$$

$$ds = \sqrt{dx^2 + dy^2}$$



- Velocity has unit speed

$$\vec{v} = \vec{\Gamma}'(s) = v\hat{t}$$

$$v = \|\vec{v}\| = 1$$

Length is trivial.
Orientation matters.

- Angle-profile representation

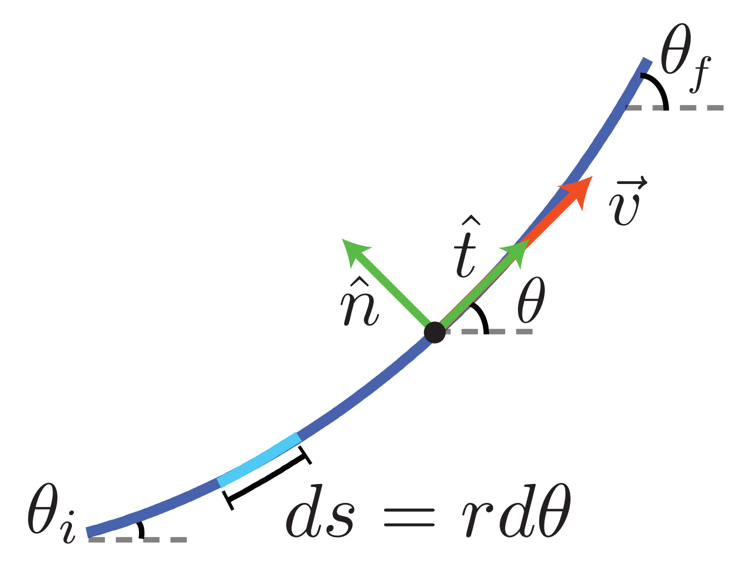
$$\{\theta(s)\}$$

Orientation of the tangent vector.

Convex curves

- Monotonic angle-profile

$$\{\theta(s)\}$$



- Angle coordinate

$$\Gamma(\theta) = (x(\theta), y(\theta))$$

- Velocity

$$\vec{v} = \vec{\Gamma}'(\theta) = v\hat{t}$$

$$v = ds/d\theta = r(\theta)$$

length = radius of curvature.
Orientation is trivially given.

$$\hat{t}(\theta) = \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$$

- Radius-profile representation

$$\{r(\theta)\}$$

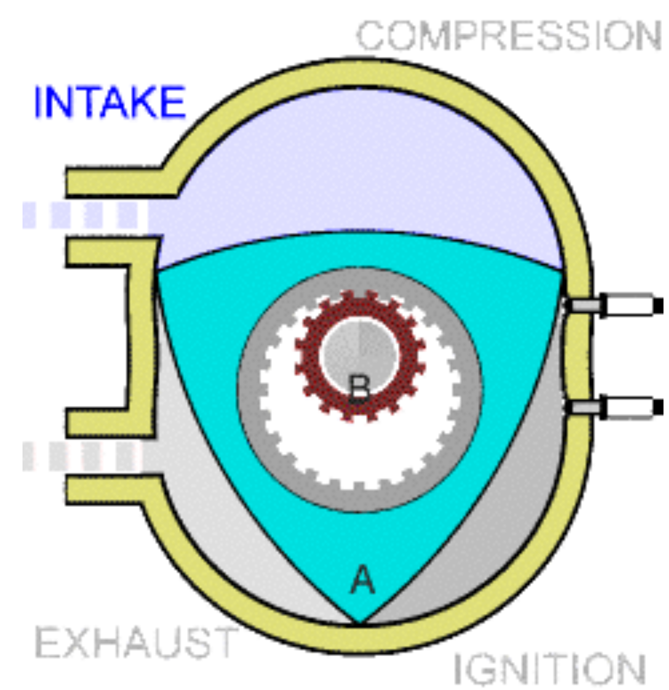
- Curve reconstruction

$$\Gamma(\theta) = \Gamma(\theta_o) + \int_{\theta_o}^{\theta} r(\theta)\hat{t}(\tilde{\theta})\, d\tilde{\theta}$$

So what about the coins?

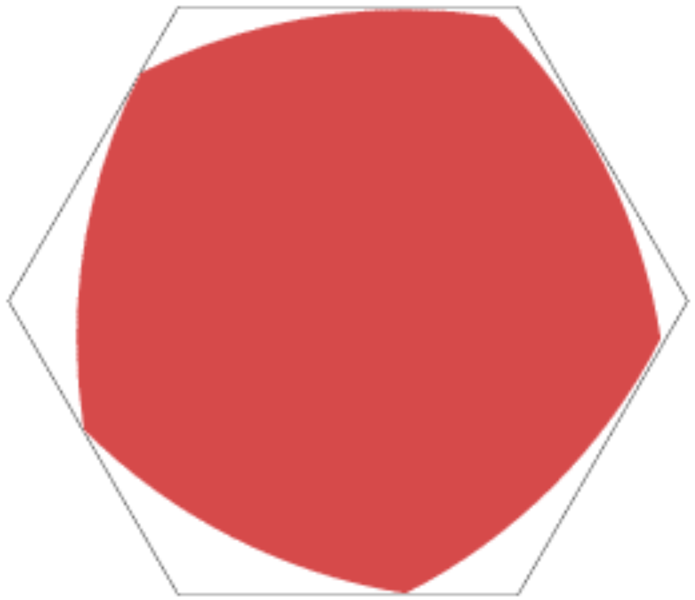
- How about this manhole?

- Wankel engine?

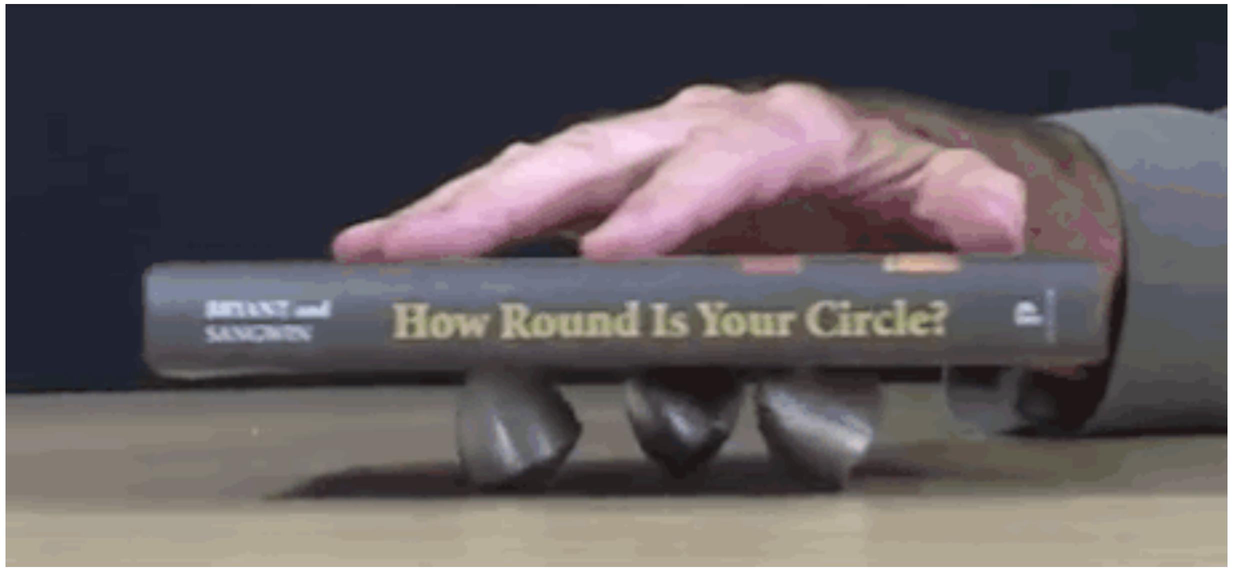


Curves of constant width

Reuleaux polygons

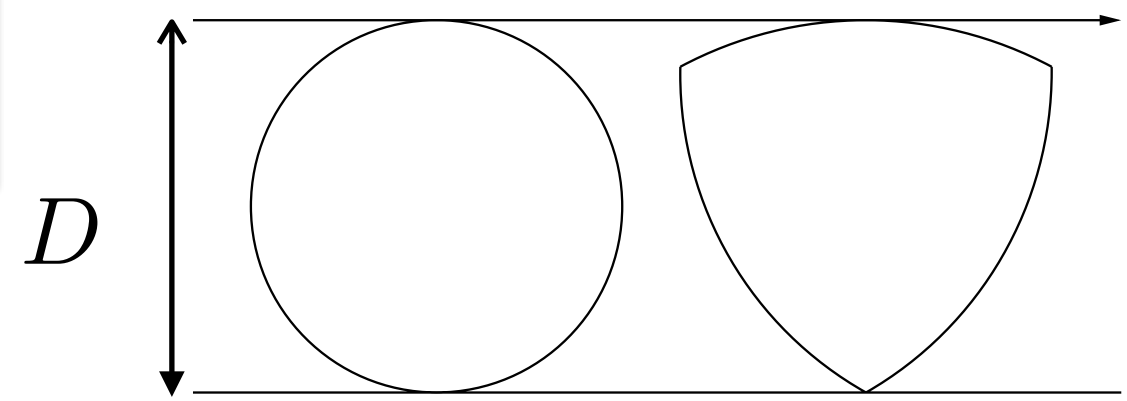


3D version



Barbier's theorem

Curves of constant width D
have circumference $D\pi$

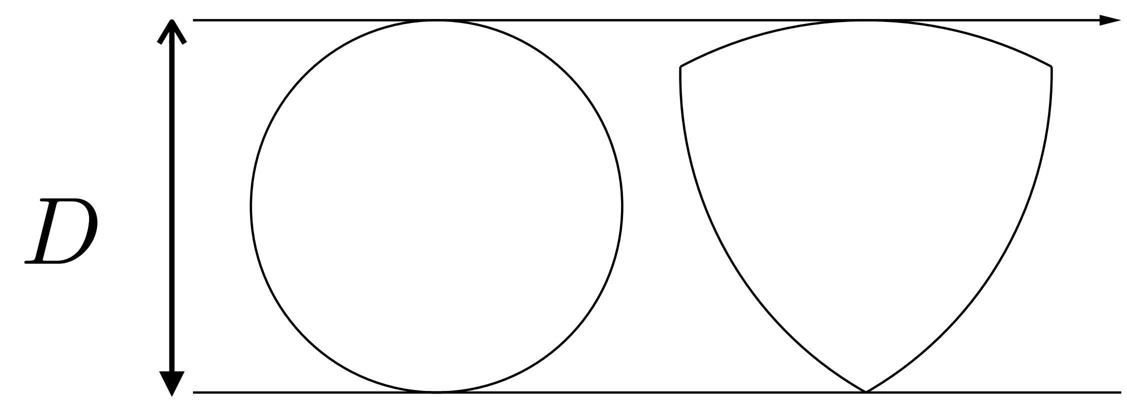


$$\int_0^{2\pi} r(\theta) d\theta = D\pi$$

proof

$$y(\pi) - y(0) = \int_0^{\pi} r(\theta + \theta_o) \sin(\theta) d\theta = D(\theta_o)$$

$$dD/d\theta_o = \int_0^{\pi} r'(\theta + \theta_o) \sin(\theta) d\theta = 0$$



$$[r(\theta + \theta_o) \sin(\theta)]_0^{\pi} - \int_0^{\pi} r(\theta + \theta_o) \cos(\theta) d\theta = 0$$

$$0 \quad - \quad (x(\pi) - x(0)) = 0$$

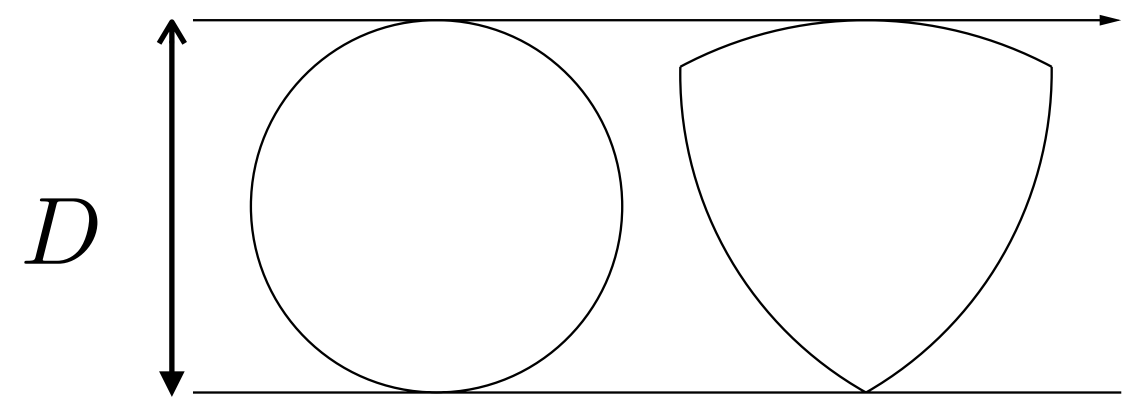
$$\therefore x(\pi) = x(0)$$

Interesting!

proof

$$x(\pi) - x(0) = \int_0^\pi r(\theta + \theta_o)\cos(\theta)d\theta = 0$$

$$\int_0^\pi r'(\theta + \theta_o)\cos(\theta)d\theta = 0$$



$$[r(\theta + \theta_o)\cos(\theta)]_0^\pi - \int_0^\pi r(\theta + \theta_o)\sin(\theta)d\theta = 0$$

$$\boxed{r(\theta) + r(\theta + \pi) = D}$$

Huh!

proof

Huh theorem? :)

$$r(\theta) + r(\theta + \pi) = D$$

↓

Barbier's theorem

$$\int_0^{2\pi} r(\theta) d\theta = D\pi$$